

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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Pearson Edexcel International GCSE

Time 2 hours

Paper
reference

4PM1/01R

Further Pure Mathematics PAPER 1R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 - *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
 - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times \text{slant height}$

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \frac{a}{1 - r}$ $|r| < 1$

Binomial series

$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{2!}x^2 + \dots + \frac{n(n - 1)\dots(n - r + 1)}{r!}x^r + \dots$ for $|x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 (a)** On the grid below, draw the graph of the line with equation

(i) $4x + 5y = 20$ (ii) $3y - 4x = -12$

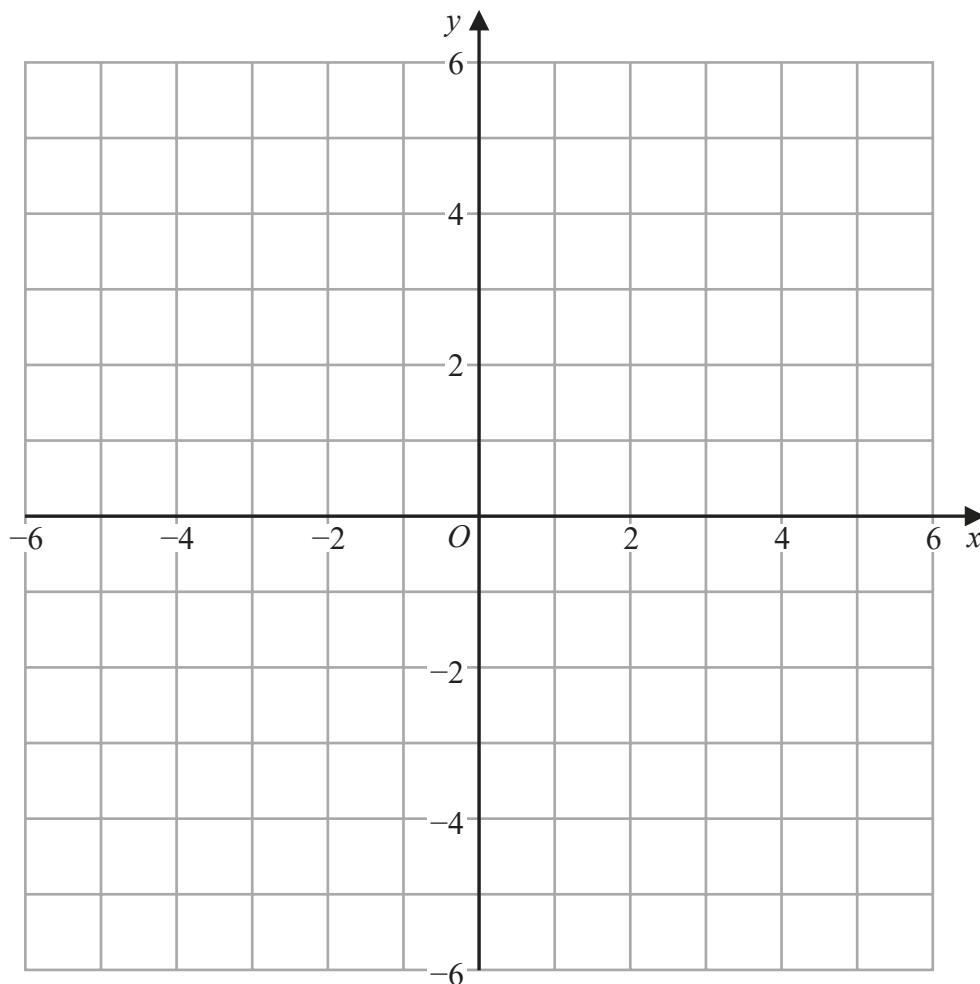
(2)

- (b)** Show, by shading on the grid, the region R defined by the inequalities

$$4x + 5y \leqslant 20 \quad 3y - 4x \geqslant -12 \quad y \leqslant 3 \quad x \geqslant 1$$

Label the region R .

(2)



Turn over for a spare grid if you need to redraw your graph.



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Question 1 continued

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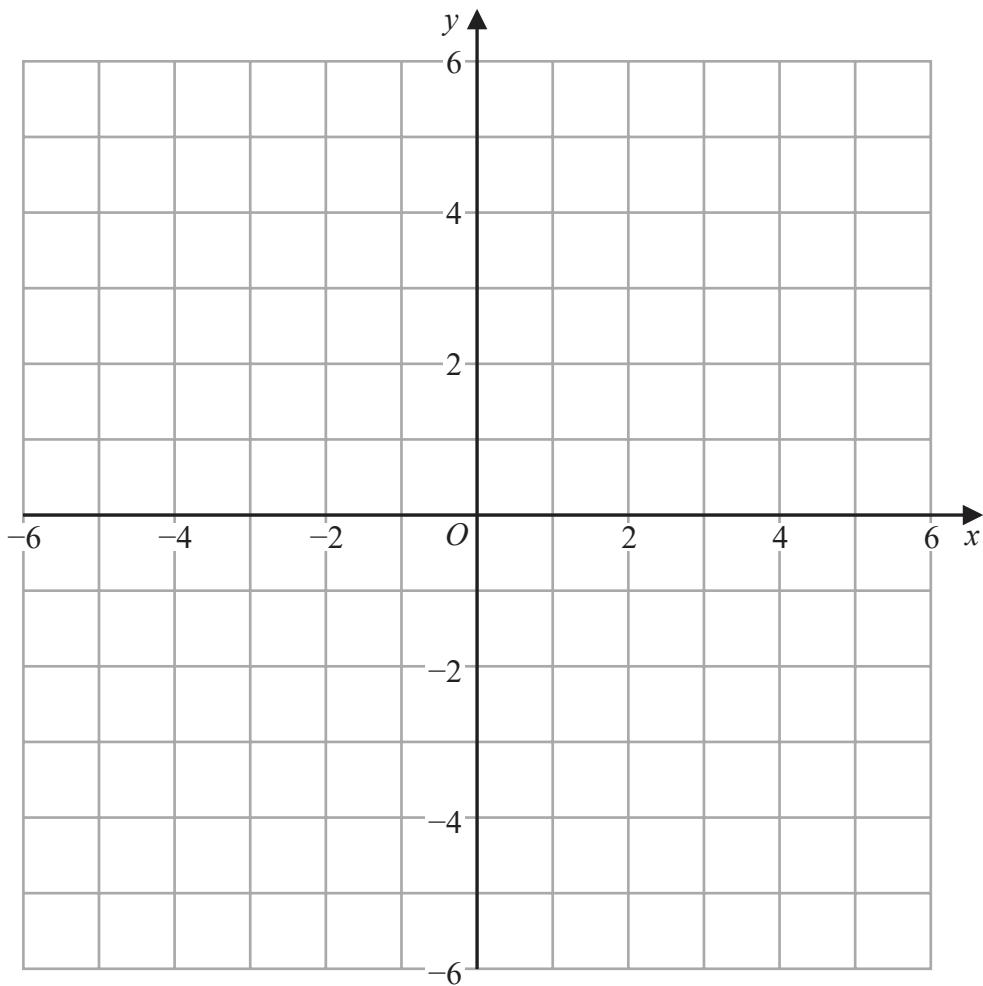
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Question 1 continued

Only use this grid if you need to redraw your graph.



(Total for Question 1 is 4 marks)



P 6 6 3 0 9 A 0 5 3 6

2 The n th term of an arithmetic series is u_n

Given that $u_5 = 46$ and that $u_{20} = 181$

(a) find

(i) the common difference of this series,

(ii) the first term of this series.

(4)

(b) Evaluate $\sum_{n=21}^{50} u_n$

(4)



Question 2 continued

(Total for Question 2 is 8 marks)



P 6 6 3 0 9 A 0 7 3 6

- 3 The point A has coordinates $(1, 7)$ and the point B has coordinates $(9, 3)$

The line l is the perpendicular bisector of AB

- (a) Find an equation of l

(5)

The line l crosses the x -axis at the point C

- (b) Find the area of the triangle ABC

(5)



Question 3 continued

(Total for Question 3 is 10 marks)



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Diagram **NOT**
accurately drawn

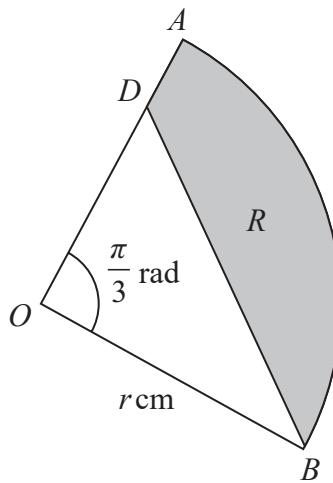


Figure 1

Figure 1 shows sector OAB of a circle with centre O and radius $r\text{ cm}$.

The angle $\angle AOB = \frac{\pi}{3}$ radians.

The point D divides OA in the ratio $3 : 1$

The area of the region R , shown shaded in Figure 1, is 79.5 cm^2

(a) Calculate the value, to 2 significant figures, of r

(4)

(b) Calculate the perimeter, in cm to 2 significant figures, of the region R

(4)

Question 4 continued

(Total for Question 4 is 8 marks)



P 6 6 3 0 9 A 0 1 1 3 6

$$f(x) = (1 + ax)^n \text{ where } n > 2$$

- (a) Write down the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3

Give each coefficient in terms of n and a

(2)

In the expansion of $f(x)$, the coefficient of x is 8 and the coefficient of x^2 is 30

- (b) Find the value of n and the value of a

(4)

- (c) Calculate the coefficient of x^3 in $f(x)$

(2)



Question 5 continued

(Total for Question 5 is 8 marks)



P 6 6 3 0 9 A 0 1 3 3 6

- 6 The points P , Q , R and S are the vertices of a quadrilateral $PQRS$ such that

$$\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} \quad \overrightarrow{PR} = -\mathbf{i} + 18\mathbf{j} \quad \overrightarrow{PS} = -3\mathbf{i} + 15\mathbf{j}$$

- (a) Show that $PQRS$ is a parallelogram.

(4)

- (b) Find a unit vector parallel to \overrightarrow{QS} as a simplified expression in terms of \mathbf{i} and \mathbf{j}

(4)

The point T lies on QS such that $QT: TS = 5:8$

- (c) Find \overrightarrow{PT} as a simplified expression in terms of \mathbf{i} and \mathbf{j}

(2)



Question 6 continued



Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)



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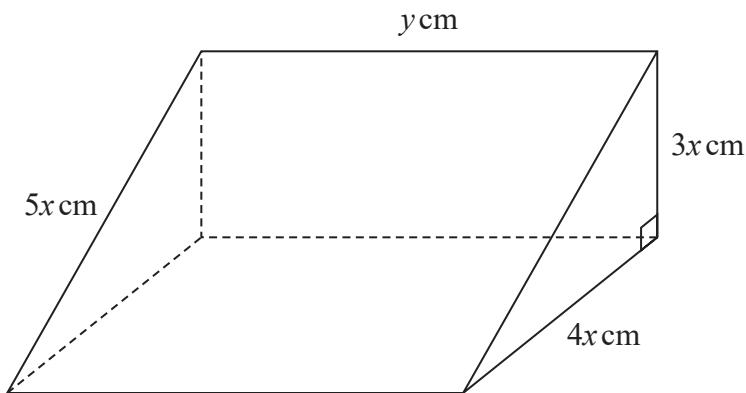


Diagram NOT
accurately drawn

Figure 2

Figure 2 shows a block of wood in the shape of a right triangular prism.

The cross section of the prism is a right-angled triangle with sides of length $3x$ cm, $4x$ cm and $5x$ cm.

The length of the prism is y cm.

The total surface area of the five faces of the prism is 144 cm^2

The volume of the prism is $V\text{ cm}^3$

(a) Show that

$$V = 72x - 6x^3 \quad (5)$$

Given that x can vary,

(b) use calculus to find the value of x for which V is a maximum, justifying that this value gives a maximum value of V

(4)

(c) Find the maximum value of V

(2)



Question 7 continued



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Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)



P 6 6 3 0 9 A 0 2 1 3 6

- 8 The curve C has equation $y = 2x^2 - \sin x$

The point A on C has x coordinate π

Show that an equation of the normal to C at the point A is

$$x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0 \quad (8)$$

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Question 8 continued

(Total for Question 8 is 8 marks)



P 6 6 3 0 9 A 0 2 3 3 6

- 9 (a) Complete the table of values for

$$y = \frac{x^3 + 4}{5 - x}$$

giving your answers to 2 decimal places.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	-0.57	0.10	0.5		0.8		1.25		4

(2)

- (b) On the grid opposite, draw the graph of

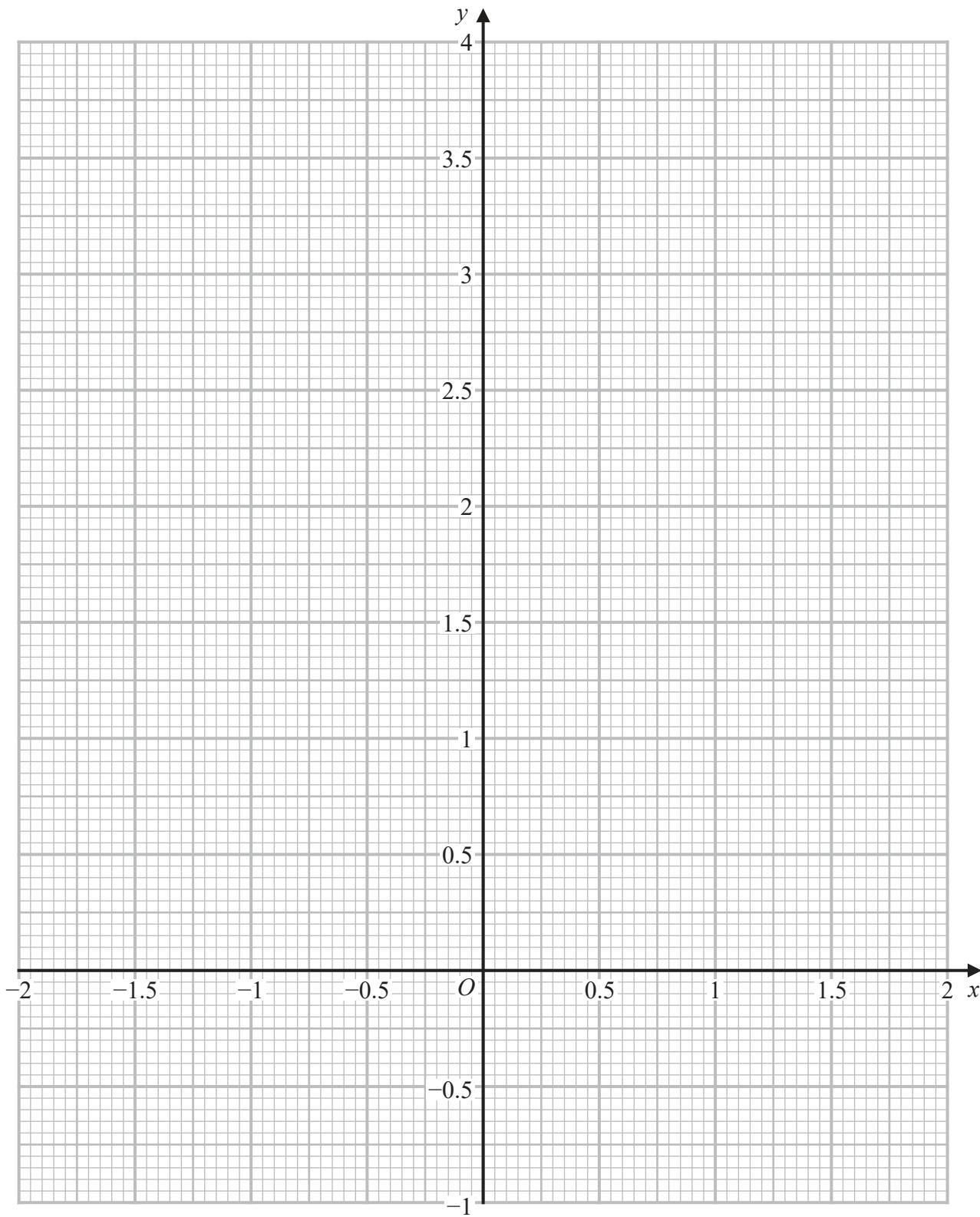
$$y = \frac{x^3 + 4}{5 - x} \quad \text{for } -2 \leq x \leq 2$$

(2)

- (c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $x^3 - x^2 + 8x - 11 = 0$ in the interval $-2 \leq x \leq 2$

(5)



Question 9 continued

Turn over for a spare grid if you need to redraw your graph.



Question 9 continued

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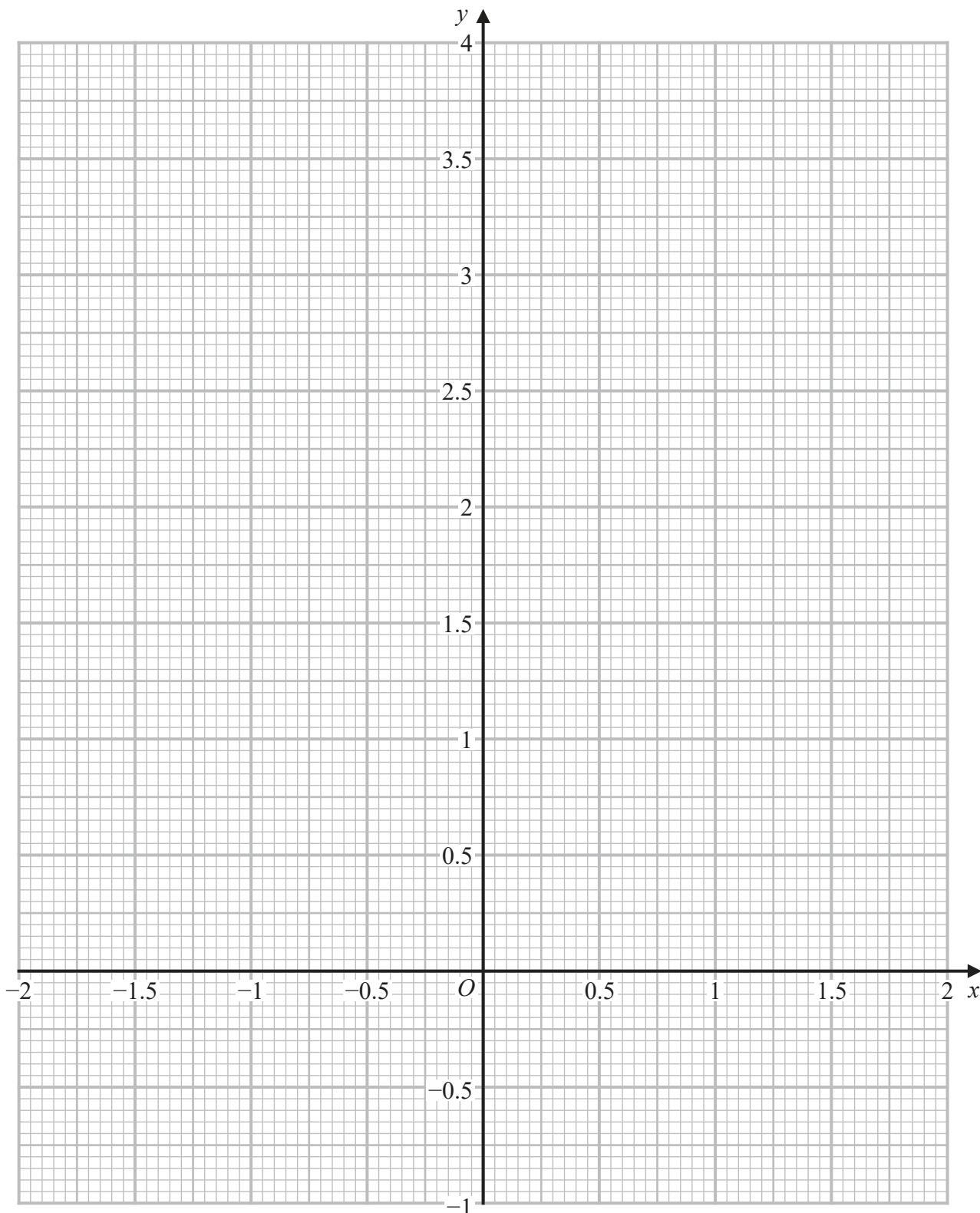
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Question 9 continued

Only use this grid if you need to redraw your graph.



(Total for Question 9 is 9 marks)



P 6 6 3 0 9 A 0 2 7 3 6

- 10 (a) Use the factor theorem to show that $(4x - 3)$ is a factor of

$$16x^3 + 11x - 15$$

(2)

- (b) Using formulae given on page 2, show that

(i) $\sin 2\theta = 2 \sin \theta \cos \theta$

(ii) $\cos 2\theta = 2 \cos^2 \theta - 1$

(5)

- (c) Show that the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

becomes the equation

$$16x^3 + 11x - 15 = 0$$

by using the substitution $x = \cos \theta$

(4)

- (d) Hence show that any solution of the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

is given by $\cos \theta = \frac{3}{4}$

(4)



Question 10 continued



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Question 10 continued

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Question 10 continued

(Total for Question 10 is 15 marks)



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11

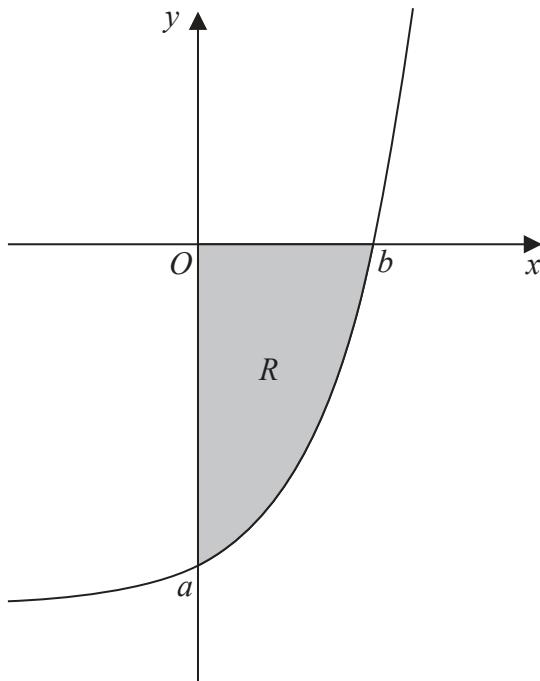


Diagram NOT
accurately drawn

Figure 3

The finite region R , shown shaded in Figure 3, is bounded by the curve with equation $y = e^{2x} - 9$ and the coordinate axes.

The curve crosses the coordinate axes at the points with coordinates $(0, a)$ and $(b, 0)$

(a) (i) Find the value of a

(ii) Show that $b = \ln 3$

(3)

The region R is rotated through 360° about the x -axis.

(b) Use calculus to find the volume of the solid generated.

Give your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers.

(6)



Question 11 continued



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Question 11 continued

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Question 11 continued



P 6 6 3 0 9 A 0 3 5 3 6

Question 11 continued

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(Total for Question 11 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS

